1906MNRAS..66...76D

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Frederick Joseph William Crowe, Organist of Chichester Cathedral, Marsden, Chichester (proposed by J. A. Greenwood) ;

Thomas Edward Heath, 53 Park Place, Cardiff (proposed by J. Larmor) ;

Samuel Thomas Johnson, Melrose, Bridport, Dorset (proposed by H. H. Turner) ;

George Tyrrell McCaw, B.Sc., Geodetic Survey, North Eastern Rhodesia, South Africa (proposed by S. S. Hough) ; and

Percy Alfred Talbot, B.A., F.R.G.S., F.A.I., Royal Societies' Club, St. James's Street, S.W. (proposed by E. A. Reeves).

Sixty-three presents were announced as having been received since the last meeting, including, amongst others :—

T. E. Heath, Our Stellar Universe, presented by the author ; Observatoire de Bordeaux, Catalogue photographique du Ciel (coordonnées rectilignes), tome i., presented by the French Government.

Astrographic Chart of the Heavens :—18 Charts from the Royal Observatory, Greenwich ; 64 Charts from the Paris Observatory, and 10 Charts from the Observatory, San Fernando.

The Value of the Constant of Refraction. By Dr. L. de Ball.

(*Communicated by the Astronomer Royal.*)

Professor Bauschinger, in his researches on astronomical refraction,* gives the following values for the constants of refraction as reduced to the height of the barometer of 760 mm. and to the temperature of 0° C. of the internal and the external thermometer, to which I add that lately determined by Dr. Courvoisier : †

60″·120	Greenwich	1857–65	60″·209	Pulkowa	1865
·192	„	1877–86	·058	„	1885
·268	Tab. Pulk.		·104	Bauschinger	
·122	Fuss (Pulkowa)		·161	Courvoisier	

---

* *Annals of the Munich Observatory*, vol. iii.

† *Publications of the Heidelberg Observatory*, vol. iii.

1906MNRAS..66...76D

At Greenwich and Pulkowa, however, only the reduction of the barometer to 0° C. has been applied, whereas Professor Bauschinger and Dr. Courvoisier have also taken into account the correction for gravitation and the hygrometric state of the atmosphere. A reduction has, therefore, to be applied to the above values deduced from the observations of Greenwich and Pulkowa before they can be compared with the values determined at Munich and Heidelberg. The determination of the values used for these reductions is the purpose of this paper.

For zenith distances less than 84° we can compute the refraction by means of the series

$$(1)\ \text{Refraction} = \frac{a}{\sin 1''}\left(a_0 \tan z - a_1 \tan^3 \phi z + \ \ldots\right)$$

where a depends on the density of the air at the place of observation, and the coefficients a_0, a_1, . . . are functions of the temperature and of a. The density of the air at the place of observation can be found when we know the height of the barometer (B), the readings of the internal (τ) and external (t) thermometer, and the vapour pressure (π). In the following pages B and π are supposed to be expressed in millimeters and the temperature in centigrades. When the vapour pressure for each time of observation is not known, but only its mean value π_0, we find for the density of the air ρ (*Monthly Notices*, 1905 June)

$$(2)\ \rho = \frac{B}{760}\,\frac{1 - 0{\cdot}0001627\tau}{1 + 0{\cdot}003663t}\left(1 - 0{\cdot}000000196h - 0{\cdot}00265 \cos 2\phi\right)\left(1 - \tfrac{3}{8}\,\frac{\pi_0}{B}\right),$$

where h = the height of the place of observation above the sea level (in meters) and φ = the geographical latitude of the place of observation.

In order to compute the value of a, which corresponds to a given ρ, we need only know the value a_0 (the so called constant of refraction), which holds for a definite but arbitrarily chosen density ρ_0. If, namely, c is an absolute constant, then a and a_0 are determined by the equations

$$(3)\ a = \frac{c\rho}{1 + 2c\rho}, \qquad a_0 = \frac{c\rho_0}{1 + 2c\rho_0}$$

whence

$$(4)\ a = \frac{\rho}{\rho_0}\,\frac{a_0}{1 - 2a_0\left(1 - \dfrac{\rho}{\rho_0}\right)}$$

When ρ_0 = the density of the air at the place of observation for B = 760 mm., $\tau = t =$ 0° C. and vapour pressure $= \pi_0$, we have according to (2)

$$(5)\ \rho_0 = \left(1 - 0{\cdot}000000196h - 0{\cdot}00265 \cos 2\phi\right)\left(1 - \tfrac{3}{8}\,\frac{\pi_0}{760}\right)$$

1906MNRAS..66...76D

From (2) and (5) it follows that

$$(6)\ \frac{\rho}{\rho_0}=\frac{B(1-0{\cdot}000162\tau)}{760(1+0{\cdot}003663t)}$$

If in equation (4) we substitute this expression and the supposed known value of a_0 which corresponds to the density ρ_0 given in (5), we obtain a ; by means of a and t the refraction is directly deduced from the equation (1). If, therefore, the constant of refraction corresponds to the density given in (5), we have to apply to the observed height of the barometer the reduction to 0° C. only. Reversely : *when to the observed height of the barometer we apply only the reduction to* 0° *C.* (Greenwich, Pulkowa), *and we derive from the observations the correction of an approximate value adopted for* a_0, *the new value of the constant of refraction will hold for the density of the air given in* (5).

If besides the height of the barometer and the temperature we also know the observed vapour pressure π, we obtain instead of the approximate value (2) of ρ its exact value when we substitute π for π_0 ; therefore we have

$$(7)\ \rho=\frac{B}{760}\,\frac{1-0{\cdot}000162\tau}{1+0{\cdot}003663t}\,(1-0{\cdot}000000196h-0{\cdot}00265\cos 2\phi)\left(1-\tfrac{3}{8}\frac{\pi}{B}\right)$$

Choosing for ρ_0 the density of the air at the sea level and at the geographical latitude of 45° for $B = 760$ mm., $\tau = t = 0$°C., $\pi = 6$ mm., we obtain

$$(8)\ \rho_0 = 1-\tfrac{3}{8}\,\tfrac{6}{760}$$

and from (7) and (8) when, for brevity, we put

$$(9)\ B\left\{1-0{\cdot}000162\tau-0{\cdot}000000196h-0{\cdot}00265\cos 2\phi+\tfrac{3}{8}\left(\tfrac{6}{760}-\frac{\pi}{B}\right)\right\}=b,$$

it follows that

$$(10)\ \frac{\rho}{\rho_0}=\frac{b}{760(1+0{\cdot}003663t)}$$

Here we may repeat—*mutatis mutandis*—the remarks made above in connection with the equation (6), and it appears that: *If instead of the observed height of the barometer B we use the barometer height b, as reduced according to the equation* (9) [Bauschinger, Courvoisier], *the value of the constant of refraction deduced from the observations holds for the density of the air given in* (8).

In order to reduce the values of the constants of refraction, determined in Greenwich and in Pulkowa, to that density of the air for which the values of Professor Bauschinger and Dr. Courvoisier hold we must put in the equation (4), instead of ρ, the

1906MNRAS..66...76D

value given in (8), and, instead of ρ_0, that given in (5). We then obtain

$$(11)\quad \alpha-\alpha_0=\alpha_0\left(0{\cdot}000000196h+0{\cdot}00265\cos 2\phi+\tfrac{3}{8}\frac{\pi_0-6}{760}\right)$$

We have for

$$\begin{array}{lllll} \text{Greenwich} & \alpha_0 = 60''{\cdot}156 \text{ (mean)}, & h = \overset{\text{m}}{47}, & 2\phi = & 102^\circ\ 57' \\ \text{Pulkowa} & \quad ,, \ = 60{\cdot}164 \quad ,, & \ ,, = 75, & \ ,, \ = & 119\ \ 33 \end{array}$$

Hence, if first we neglect the member depending on π_0,

Greenwich	$\alpha-\alpha_0 = -0''{\cdot}035$
Pulkowa	,, $= -0{\cdot}078$

Therefore we obtain for *the value of the constant of refraction reduced to the density* $\rho_0 = 1-\frac{3}{8}\,\frac{6}{760}$, *and hence to* $h=0$, $\phi=45^\circ$, $B = 760$ mm., $\tau = t = 0^\circ$ C., $\pi = 6$ mm.

Greenwich	$60''{\cdot}121 + 0''{\cdot}030(\pi_0-6)$
Pulkowa	$60{\cdot}086 + 0{\cdot}030(\pi_0-6)$
Munich	$60{\cdot}104$
Heidelberg	$60{\cdot}161$

If the observations were equally distributed over the whole year we might accept for π_0 the mean from the monthly means of the vapour pressure of Greenwich and Pulkowa respectively, but this assumption on the distribution of the observations is not permissible. However, even if π_0 were known, the correction depending on π_0 would be uncertain, for it is possible that in the expression for the correction mentioned $\frac{1}{8}$ has to be put in the place of $\frac{3}{8}$ (*Monthly Notices*, 1905 June).

Vienna-Ottakring: 1905 *November* 13.

Note on an Experiment on Lateral Refraction.

By B. F. E. Keeling, Survey Department, Egypt.

(*Communicated by Professor H. H. Turner.*)

A geodetic triangulation is about to be made by the Egyptian Survey Department with the object of linking up the South African survey along the 30° E. meridian and the Russian triangulation. The first question which was raised was the position of the chain, whether to utilise in the main the Nile Valley or to take the chain wholly to east or west of it. From the point of view of convenience and economy the former is evidently to be preferred, but it was asked whether it would not